**A study on baryon spectroscopy using a modified Cornel potential form**

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**Abstract**

In this work a non-relativistic three body Schrödinger equation with a simple phenomenological interacting potential form is solved numerically to obtain the resonance masses of N, Δ, Λ and $Σ$ baryons. A modified Cornell potential form is used to describe the quark-quark interacting forces inside baryon. The present simple phenomenological potential model contains only three free parameters beside the quark masses. A comparison between the present calculations and both the allowed experimental and previous theoretical results is investigated. From this study one can conclude that; the present potential form gives a considerable success in case of explaining the low-mass baryon excitations spectrum.

**Introduction**

In recent years, there is an interest in the spectroscopy of hadrons to study the hadron structures and their interacting forces. The quark model [1, 2] is used to describe the hadrons spectroscopy. Many researchers calculated the resonance mass of some baryons. An analytical solution was introduced in case of harmonic and anharmonic potential forms, using an ansatz functions, for a system consists of three identical particles [3, 4]. A numerical solution was provided for Cornel potential form in case of N and Δ baryons [5-10]. Also a numerical solution for a harmonic oscillator potential was provided for a system consists of three identical particles [11]. In the present work we construct a modified Cornel potential form which will be solved numerically by using the Jacobi method to calculate the resonance state masses of N, Δ, Λ and $Σ$ baryons. These baryons consist of identical or non-identical quarks. A comparison between these results and both the experimental and theoretical previous results will be discussed.

**The Used Model**

 Schrodinger equation for a system consists of three particles is given by:

$ \left[-\frac{ℏ^{2}}{2m\_{1}}\left.∇\_{r\_{1}}^{2}-\frac{ℏ^{2}}{2m\_{2}}∇\_{r\_{2}}^{2}-\frac{ℏ^{2}}{2m\_{3}}∇\_{r\_{3}}^{2}\right.+V\_{12}\left(r\_{12}\right)+V\_{23}\left(r\_{23}\right)+V\_{31}(r\_{31}) \right]Ψ \left( r\_{1 },r\_{2},r\_{3}\right)=EΨ \left( r\_{1 },r\_{2},r\_{3}\right) $(1)

Where$ m\_{1},m\_{2} and m\_{3} are the three quarks masses , \vec{r\_{1}},\vec{r\_{2}}and \vec{r\_{3}} are $ shown in fig (1).

3

$$→$$

$$→$$

$$→$$

0

1

2

 Fig. (1) Vector diagram of the three quarks inside the baryon.

We have to separate the center of mass from the relative coordinates using the Jacobi coordinates which are defined as [8, 11]:

$\vec{R}=\frac{m\_{1}\vec{r}\_{1}+m\_{2}\vec{r}\_{2}+m\_{3}\vec{r}\_{3}}{M},\vec{ρ}$=$c\_{1}\left(\vec{r}\_{1}-\vec{r}\_{2}\right)and \vec{λ}=c\_{2}\left(\frac{m\_{1}\vec{r}\_{1}+m\_{2}\vec{r}\_{2}}{M^{\acute{'}}}-\vec{r}\_{3}\right)$ (2)

Where M and $M^{\acute{'}} $represent the total mass and the sum of masses of particle 1 and 2 respectively. $c\_{1} $and $c\_{2}$ are constants.

Equation (1) can be rewritten in the following form using natural units $ ℏ=C=1$ and letting the coefficients of $∇\_{ρ }^{2}and ∇\_{λ}^{2}$ be equals to A:

$-\frac{1}{2M}\left.∇\_{R}^{2}\right. Ψ\_{C.M}(R)=E\_{1}Ψ\_{C.M}(R)$(3)

$\left[\left.-A(∇\_{ρ}^{2}+∇\_{λ}^{2})\right.+V(x)\right]Ψ\_{r}(x) =E\_{2}Ψ\_{r}(x) $ (4)

Where $x$ =$\sqrt{ρ^{2}+λ^{2}}$ is the hyper radius,$ E\_{1}+E\_{2}=E$

And A=$ \frac{c\_{1}^{2}}{2μ\_{12}}=\frac{c\_{2}^{2}}{2μ\_{12,3}}$ .

 $Ψ\_{C.M}(R)$ and $Ψ\_{r}(x)$ represent the center of mass and the relative wave functions respectively.

Let $E\_{1}=zero ,$ then $E\_{2}$ must be equals to E.

The three body wave function can be written as [5, 12, 13]:

$Ψ\_{r} ( \vec{ρ,} \vec{λ})$=$\sum\_{γ}^{}Ψ\_{γ}(x)Y\_{γ}\left(Ω\right)$(5)

Where $γ$ and $Y\_{γ}\left(Ω\right)$are the grand orbital quantum number and the hyper spherical harmonic respectively.

If the potential $V(x)$is assumed to depend only on the hyper radius $x$ ,the space wave function is factorized similarly to the central potential [3,5] and only one term in equation (5) is a solution of the relative Schrodinger equation.

 Therefore equation (4) can be written in the following form:

$\frac{d^{2}Ψ\_{γ}(x) }{dx^{2}}+\frac{D-1}{x}\frac{dΨ\_{γ}(x) }{dx}-\frac{L\_{\left(Ω\right) }^{2}}{x^{2}}Ψ\_{γ}(x)+\frac{1}{A}\left(E -V (x) \right)Ψ\_{γ}(x)=0$ $ (6)$

Where D represents the dimension of the $\rightharpoonaccent{x }$ and $L\_{\left(Ω\right) }^{2}$ is the angular momentum operator whose Eigen functions are [3, 6, 8]$:$

$L\_{\left(Ω\right) }^{2}Y\_{\left[γ\right]}=-γ\left(γ+D-2\right)Y\_{\left[γ\right]}$ (7)

Where $γ$=0,1,2,…..

The assumed potential model is given as:

$V \left(x\right)=α\_{1}x-\frac{α\_{2}}{x}EXP\left(-α\_{3}x\right)$ (8)

Where $α\_{1}$,$ α\_{2}$ and $α\_{3}$ are free parameters.

Now, applying the following transformations:

$Ψ\_{γ}(x) =u\_{γ}(x) /x^{\frac{\left(D-1\right)}{2}}$ (9) If one carries the first and second derivatives of$ Ψ\_{γ}(x) $, equation (6) takes the following form:

$\left[\frac{d^{2} }{dx^{2}}+\frac{1}{A}\left(E –V \left(x\right)-A\left(\frac{\left(D-1\right)\left(D-3\right)}{4x^{2}}+\frac{γ\left(γ+D-2\right)}{x^{2}}\right)\right) \right]u\_{γ}(x)=0$ (10)

With the boundary conditions:

$u\_{γ}\left(0\right)=u\_{γ}\left(\infty \right)$ (11)

Substituting in equation (10) with:

$λ=\frac{E}{A}$ and φ ($x$) =$ \frac{1}{A} V \left(x\right)+\frac{\left(D-1\right)\left(D-3\right)}{4x^{2}}+\frac{γ\left(γ+D-2\right)}{x^{2}} $ (12)

Equation (10) becomes in the following form:

$\frac{d^{2}u\_{γ}(x) }{dx^{2}}+\left(λ-φ \left(x\right)\right)u\_{γ}(x)=0 $ (13)

Let us define the dimensionless variables g and$ ϴ \left(g\right)$, to transform the range of $x from \left(\infty ,0\right) to \left(0,1\right)as:$

g=$\frac{1}{1+\frac{x}{x\_{0}}}$ , $x\_{0}=1Gev^{-1} and ϴ \left(g\right)$=g$ϴ$(g) (14)

Introducing these variables in equation (13), one gets:

$\frac{d^{2}ϴ \left(g\right) }{dg^{2}}+\left[\frac{x\_{0}^{4}}{g^{4}}\right]\left(λ-φ \left(g\right)\right)ϴ \left(g\right)=0 $ (15)

With the boundary conditions:

$ϴ \left(0\right)$=$ϴ \left(1\right)$=0 (16)

To transform equation (15) into a matrix equation, the range of (g) from zero to one can be divided into (n+2) points with h interval; and labeled by subscript (J). Now, the boundary conditions at J=0 and n+1 are:

$ϴ\_{n+1}=ϴ\_{0}$ (17)

By using the finite difference approximation $\left[14\right]:$

$\frac{d^{2}ϴ \left(g\right) }{dg^{2}}=\frac{1}{12h^{2}}\left[-ϴ\_{J-2}+16ϴ\_{J-1}-30ϴ\_{J}+16ϴ\_{J+1}-ϴ\_{J+2}\right]+O(h^{4})$ (18)

Where the term $O(h^{4})$ represents the expected error in equation (18).

Where,

$ϴ\_{-1}$=$\left(-1\right)^{γ}ϴ\_{1}$+$ O\left(h^{2}\right);$ (19)

$ϴ\_{n+2}$=$\left(-1\right)^{γ+1}ϴ\_{n}$+$ O\left(h^{3}\right);$ (20)

And substituting form equation (18) into equation (15), one gets:

 $\left(ϴ\_{J-2}-16ϴ\_{J-1}+30ϴ\_{J}-16ϴ\_{J+1}+ϴ\_{J+2}\right)+\frac{12h^{2}}{\left(Jh\right)^{4}}\left(φ \left(Jh\right)-λ\right)ϴ\_{J}=0$ (21)

These linear equations in $ϴ\_{J}$ can be written in the following matrix equation:

$\left(A-λI\right)ϴ=0$ (22)

This true eigen value equation is solved numerically using Jacobi method [15, 16] to obtain both eigen values and eigen vectors. The resonance mass; M; of the state is related to the eigen value; E; (non-relativistic) by the following relation;

M=$m\_{1}+m\_{2}+m\_{3}+E$ (23)

**Results and Discussions**

 In the present work the resonance masses of the considered baryons are calculated using a modified Cornell potential form (see equation (8)).

The quark masses and the potential parameter values are adjusted to reproduce the resonance masses of the considered baryons (N, Δ , Λ and $Σ$ ).

The parameter values are determined through $χ^{2}$-test [17],

$χ^{2}=\frac{1}{N}\sqrt{\sum\_{i=1}^{N}\left(\frac{\left(M\_{i}^{theo}-M\_{i}^{exp}\right)}{e\_{i}}\right)^{2}}$ (24)

Where $e\_{i}$represent the experimental error in the$ i^{th}$ state.

The parameters used in the present calculations are given in Table (1). A comparison between the behavior of the present potential form and the Cornel potential Ref. [7] is shown in fig.(2).

Fig. (2): Comparison between the behavior of the present potential and

 Cornell potential forms [7].

The calculated resonance masses of N baryons in comparison with the experimental data [18] and previous works [5, 7, 9] are given in Table (2). From this table one notices that, the previous works calculate 4-states only while there are nine experimental states. All allowed experimental states are calculated using the present potential form and give a better agreement with the experimental data than the previous theoretical results of [5, 7, 9].

The calculated resonance masses of Δ baryons in comparison to the allowed experimental data and the previous theoretical works are given in Table (3). Also the calculated resonance masses using the present potential form give more agreement with the experimental data.

The calculated resonance masses of Λ baryons in comparison to the experimental data are given in Table (4). The present potential calculations give a satisfied agreement with the allowed experimental data.

Finally, the calculated resonance masses of $Σ$ baryons using the present potential form in comparison to the experimental data are given in Table (5).

From this study one can conclude that, the suggested potential form can be used to study the resonance masses of all considered baryons. The calculated resonance masses using this simple phenomenological potential form give a satisfied agreement with the experimental data than those of the previous theoretical works in case of all considered baryons.

Table 1: Parameter values of the considered potential model.

|  |  |
| --- | --- |
| parameters | values |
| $$α\_{1}$$ | 0.00575±0.000575$ Gev^{2}$ |
| $$α\_{2}$$ | 6.39±0.639 |
| $$α\_{3}$$ | 0.03±0.003$ Gev$ |
| $$m\_{u}$$ | 0.448±0.052$ Gev$ |
| $$m\_{d}$$ | 0.7225±0.1225$ Gev$ |
| $$m\_{s}$$ | 0.9205±0.0205$ Gev$ |
| A | 1.273±0.1273 $ Gev^{-1}$ |

Table 2: Calculated resonance masses of N baryon in comparison to the

 experimental data and previous works.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Baryonstate | Status | $$M\_{exp}$$Ref [18](Mev) | $$γ$$ | Ref [5](Mev) | Ref [7](Mev) | Ref [9](Mev) | Presentform(Mev) |
| N(938)P11 | \*\*\*\* | 938.272013±0.000023 | 0 | 938 | 938 | 938 | 938 |
| N(1440)P11 | \*\*\*\* | 1445±25 | 1562 | 1463 | 1557 | 1452 |
| N(1710)P11 | \*\*\* | 1710±30 | 1819 | 1752 | 1807 | 1674 |
| N(1535)S11 | \*\*\*\* | 1535±10 | 1 | 1545 | 1524 | 1543 | 1540 |
| $$χ^{2}$$ | - | - | 1.5 | 0.5 | 1.4 | 0.3 |
| N(1905)S11 | \*\* | 1905 | - | - | - | 1899 |
| N(2090)S11 | \* | 2150±50 | - | - | - | 2086 |
| N(1900)P13 | \*\* | 1900 | 2 | - | - | - | 1900 |
| N(2190)G17 | \*\*\*\* | 2150±50 | 3 | - | - | - | 2109 |
| N(2220)H19 | \*\*\*\* | 2225±25 | 4 | - | - | - | 2184 |
| $$χ^{2}$$ | - | - | - | - | - | - | 0.29 |

Table 3: Calculated resonance masses of Δ baryon in comparison to the

 experimental data and previous works.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Baryonstate | Status | $$M\_{exp}$$Ref [18](Mev) | $$γ$$ | Ref [5](Mev) | Ref [7](Mev) | Ref[9](Mev) | Presentform(Mev) |
| Δ (1232)P33 | \*\*\*\* | 1232±1 | 0 | 1237 | 1232 | 1237 | 1232 |
| Δ (1600)P33 | \*\*\* | 1625±75 | 1670 | 1727 | 1675 | 1706 |
| Δ (1620)S31 | \*\*\*\* | 1630±30 | 1 | 1546 | 1573 | 1544 | 1624 |
| Δ (1900)S31 | \*\* | 1900 | - | - | - | 1900 |
| Δ (2150)S31 | \* | 2150 | - | - | - | 2050 |
| Δ (1910)P31 | \*\*\*\* | 1895±25 | 2 | 1850 | 1953 | 1863 | 1910 |
| $$χ^{2}$$ | - | - | - | 0.4 | 0.6 | 1.5 | 0.3 |
| )Δ 2200)G37 | \* | 2200 | 3 | - | - | - | 2200 |
| )Δ 2400)G39 | \*\* | 2400 | - | - | - | 2323 |
| $$χ^{2}$$ |  |  |  | - | - | - | 0.17 |

 Table 4: Calculated resonance masses of Λ baryon in comparison to

 the experimental data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Baryonstate | Status | $M\_{exp}$ Ref [18] (Mev) | $$γ$$ | Presentform(Mev) |
| Λ(1116)P01 | \*\*\*\* | 1115.683±0.006 | 0 | 1116 |
| Λ(1600)P01 | \*\*\* | 1630±70 | 1599 |
| Λ(1810)P01 | \*\*\* | 1800±50 | 1810 |
| Λ(1670)S01 | \*\*\*\* | 1670±10 | 1 | 1662 |
| Λ(1800)S01 | \*\*\* | 1785±65 | 1871 |
| Λ(1890)P03 | \*\*\*\* | 1880±30 | 2 | 1890 |
| $$χ^{2}$$ | - | - | - | 0.27 |

 Table 5: Calculated resonance masses of $Σ$ baryon in comparison to

 the experimental data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| .Baryonstate | Status | $$M\_{exp}$$Ref [18] (Mev) | $$γ$$ | Presentform(Mev) |
| $Σ$(1193)P11 | \*\*\*\* | 1192.642±0.024 | 0 | 1193 |
| $Σ$(1660)P11 | \*\*\* | 1660±30 | 1667 |
| $Σ$(1880)P11 | \*\* | 1880 | 1872 |
| $Σ$(1620)S11 | \*\* | 1620 | 1 | 1620 |
| $Σ$(1750)S11 | \*\*\* | 1765±35 | 1824 |
| $$χ^{2}$$ | - | - | - | 0.34 |

**References**

[1] C. Amsler, T. DeGrand and B. Krusche, *Physics Letters*, **B667**, 1 (2008).

[2] B. Metsch, Arxive :hep-ph/0403118v1 11 Mar 2004.

[3] A. A. Rajabi, *Iranian Journal of Physics Research*, **5**, No.2, 37 (2005).

[4] A. A. Rajabi, *Few-body Systems*, **37**,197 (2005).

[5] E. Santopinto, F. Iachello and M. M. Giannini, *Nuclear Physics,* **A623** 100c (1997).

[6] M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto and L. Tiator, *Physics Letters,* **B364**, 231 (1995).

[7] M. M. Giannini, E. Santopinto, and A. Vassallo, *The European Physical Journal*, **A12**, 447 (2001).

[8] I. M. Narodetskii and M. A. Trusov, *Nuclear Physics B* (Proc. Suppl.) **115**, 20 (2003).

[9] E. Santopinto, F. Iachello and M. M. Giannini, *The European Physical Journal*, **A1**, 307 (1998).

[10] M. M. Giannini, E. Santopinto, and A. Vassallo, *Progress in Particle and Nuclear Physics*, **50,** 263 (2003).

[11] E. Cuervo-Reyes, M. Rigol and J. Rubayo-Soneira, *Revista Brasileira de Ensino de Fisica*, **25**, No.1, 18 (2003).

[12] J. S. Avery, *Journal of Computational and Applied Mathematics*, **233**, 1366 (2010).

[13] J. L. Ballot, and M. F. La Ripelle, *Annals of Physics,* **127**, 62 (1980).

[14] M. Abramomwitiz and I. A. Stegun, “*Handbook of Math. Functions*” National Bureau of Standards U. S. Gpo, (1972).

[15] J. H. Mathews and K. K. Fink,"*Numerical Methods Using Matlab*", 4th Edition, (2004).

[16] A. R. Goulary and G. A. Watson "*Computational Methods for Matrix Eigen Problems*", John Wiley, (1973).

# [17] T. D. V Swinscow, "*Statistics at Square One*", **BMJ Publishing Group**, Ninth Edition (**1997**).

[18] K. Nakamura *et al.,* (*Particle Data Group*), JP G **37**, 075021 (2010).